Mixed Scattering Matrix: Properties and Applications

Abstract

The lecture provides a comprehensive overview of the mixed scattering matrix (P-matrix) theory. Properties of the admittance, wave scattering, and mixed scattering matrices of the arbitrary acoustoelectric multiport network are discussed where the mixed scattering matrix **M** is defined as a mixed units hybrid of the scattering matrix **S** and admittance matrix **Y**. Based on the relationship between acoustic and electric variables, the equations for conversion between admittance, scattering, and mixed scattering matrices are deduced. Matrix implications due to the reciprocity and power conservation are discussed.

The general results and equations are applied to SAW transducer modeling where a conventional unapodized SAW transducer is considered as a reciprocal and lossless three-port acoustoelectric network, with two acoustic and one electric ports. Properties and physical meaning of the matrix blocks and elements are discussed. The number of the independent P-matrix elements is determined and their physical meaning is explained. Conversion between the mixed scattering matrix **M**, mixed transmission matrix **T**, and wave scattering matrix **S** is considered. It is shown that in general case of a reciprocal and lossless SAW transducer the elements of the mixed scattering matrix must satisfy a self-consistent system of equations following from the reciprocity and power conservation.

As a particular case, the mixed scattering matrix of a SAW transducer is deduced in the quasi-static approximation where a short-circuit SAW transducer is supposed to be non-reflective. In practice, the quasi-static approximation is valid if the central frequency f_0 *f*o of a SAW transducer is far away from the synchronous frequency $f_s = v / 2p$ where *v* is effective SAW velocity and *p* is the transducer period (pitch). In this case, we can neglect the interelectrode reflections and the mixed scattering matrix takes the simplest form. The general relationship between the mixed scattering matrix **M** and transmission matrix **T** is deduced for a SAW transducer. An important particular case of the conversion between scattering and transmission matrices in the quasi-static approximation completes consideration.

Contents

Admittance, wave Scattering, and mixed scattering matrices of the multi-port network

Definition of the mixed scattering matrix

- Generalized wave amplitudes and electric variables
- Conversion between admittance, wave scattering, and Mixed scattering matrices
- Reciprocity and power conservation

Mixed scattering matrix of a SAW transducer

- Three-port representation of a SAW transducer
- Physical meaning of the mixed scattering matrix
- Mixed scattering matrix terms and their physical meaning
- Conversion to the wave scattering matrix
- Properties of the reciprocal and lossless SAW transducer
- Conversion between mixed scattering and transmission matrices

SAW transducer modeling in the quasi-static approximation

- Mixed scattering matrix
- Transmission matrix

Conclusions